HEAT EXCHANGE IN GEOTHERMAL STRATA MAINTAINED BY LINEAR AND RING BATTERIES OF WELLS

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The temperature mode of geothermal saturated strata is determined taking into account the thermal currents from the surrounding mass.

In [1] an effective method was proposed for extracting geothermal heat based on the production of forced underground circulation by additional pumping of water into permeable water-saturated strata (Fig. 1). The extraction of heat occurs in this case both due to displacement of the hot water initially contained in the stratum and by extraction of the heat from the surrounding rock strata. In [1] the heat exchange is calculated for a one-dimensional scheme of the flow of water from forcing wells to operational wells assuming constancy of the temperature of the surrounding rock, which may lead to a considerable rise in the temperature of the water at the output from the stratum. Solutions for certain cases of two-dimensional flow ignoring thermal currents from the surrounding rock were obtained in [2, 3]. Below we describe an analytical method of calculating the thermal characteristics of a geothermal system for linear and ring batteries of wells (Fig. 2).

When solving the problem we will make the following basic assumption: 1) The motion of the water in the stratum has a two-dimensional character – i.e., it is independent of the vertical coordinate z and is described by Darcy's law; 2) the rate of interphase heat transfer between the enclosing rock stratum and the water is infinite; 3) the temperature across the stratum does not change, and the thermal conductivity in the longitudinal directions x and y is ignored both in the stratum and in the nonpermeable mass (the so-called Lover scheme); 4) the physical properties of the stratum, the rock, and the water are constant. With these assumptions, the system of equations of filtering and steady-state heat transfer in dimensionless form has the form

$$\bar{u}_x = \frac{\partial\bar{\psi}}{\partial\bar{y}} = -\frac{\partial\bar{p}}{\partial\bar{x}}, \quad \bar{u}_y = -\frac{\partial\bar{\psi}}{\partial\bar{x}} = -\frac{\partial\bar{p}}{\partial\bar{y}}, \tag{1}$$

$$\frac{\partial^2\bar{p}}{\partial\bar{x}^2} + \frac{\partial^2\bar{p}}{\partial\bar{y}^2} = 0, \quad \frac{\partial^2\bar{\psi}}{\partial\bar{x}^2} + \frac{\partial^2\bar{\psi}}{\partial\bar{y}^2} = 0, \tag{2}$$

$$\frac{\partial\theta}{\partial\bar{t}} + Cu\frac{\partial\theta}{\partial s} = \frac{2}{\mathrm{Pe_s}}\left.\frac{\partial\bar{T}}{\partial\bar{z}}\right|_{\bar{z}=0}, \tag{3}$$

$$\mathrm{Pe_m}\frac{\partial\bar{T}}{\partial\bar{t}} = \frac{\partial^2\bar{T}}{\partial\bar{z}^2}. \tag{4}$$

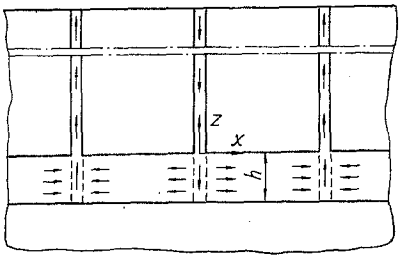

Fig. 1. Sketch showing the use of the stratum.

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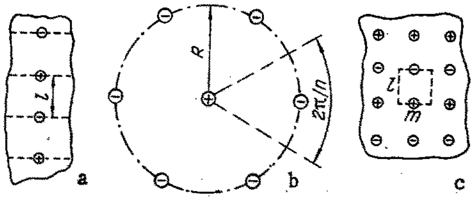

Fig. 2. Sketch showing the position of the wells.

Equation (3) is the expression for the heat balance for the stratum written along the current lines, in which the coefficient $c = \rho_w c_w/(m\rho_w c_w + (1-m)\rho_s c_s)$ represents the ratio of the velocities of propagation of thermal convective and filtering fronts. To solve Eqs. (3) and (4) we will assume that the surrounding rock mass is unlimited and has the same initial temperature T_i, while the temperature of the cooling water flowing through the pumping wells T_0 does not change with time. Then the initial and boundary conditions for Eqs. (3) and (4) take the form

$$\overline{T}(\overline{t}=0)=1,\quad \overline{T}(\overline{z}\to\infty)=1,\quad \overline{T}(\overline{z}=0)=\theta,\quad \theta(s=s_i)=0. \tag{5}$$

Hence, the system of equations (3) and (4) with condition (5) represent the Lover problem [4], formulated for individual current lines between which there is no thermal interaction. The solution of this problem is

$$\overline{T}=1-\eta\left(\overline{t}-\frac{1}{C}\int_{s_i}^{s}\frac{ds}{u}\right)\operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}_m}\,\overline{z}+\dfrac{2\sqrt{\mathrm{Pe}_m}}{\mathrm{Pe}_s}\displaystyle\int_{s_i}^{s}\frac{ds}{u}}{2\sqrt{\overline{t}-\dfrac{1}{C}\displaystyle\int_{s_i}^{s}\frac{ds}{u}}}\right), \tag{6}$$

where $\frac{1}{C}\int_{s_i}^{s}\frac{ds}{u}$ is the time of motion of the thermal front along the current line from the point where the pumping well is situated (the input of water into the stratum) s_i.

From (6) we obtain an expression for the temperature of the water at the point of output from the stratum

$$\overline{\theta}(\overline{\psi})=1-\eta(\overline{t}-\tau(\overline{\psi}))\operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}_m}}{\mathrm{Pe}_k}\,\frac{\tau(\overline{\psi})}{\sqrt{\overline{t}-\tau(\overline{\psi})}}\right), \tag{7}$$

where $\tau(\overline{\psi})=\frac{1}{C}\int_{s_i}^{s_k}\frac{ds}{u}$ is the time of motion of the thermal front from the pumping to the using well along current lines $\overline{\psi}$ = const.

The mean calorimeter temperature at the exit from the strata is the average of expression (7) over all the current lines. Consequently, the problem of finding the water temperature at the exit from the strata reduces to calculating the time of motion of the thermal convection front along the different trajectories and subsequent summation over all the trajectories.

To determine $\tau(\overline{\psi})$ we will use the following method [5]: the plane of flow $W = \overline{x} + i\overline{y}$ is conformally mapped into the upper half plane $\mathrm{Im}\,\omega \geq 0$ in such a way that the pumping well is transferred to the origin of coordinates and the operating well to infinity. In this way the isobars are converted into semicircles and the current lines into a beam of straight lines emerging from the origin of coordinates,

$$\overline{p}=\mathrm{const}-\frac{\ln r}{\pi},\quad \overline{\psi}=\frac{1}{2}-\frac{\chi}{\pi}, \tag{8}$$

and the evaluation of the curvilinear integral $\tau(\overline{\psi})$ in the W plane is replaced by an integral along the straight current lines in the plane

$$\tau(\overline{\psi})=\tau(\chi)=\frac{\pi}{C}\int_0^{\infty}\left|\frac{dW}{d\omega}\right|^2 r\,dr. \tag{9}$$

Note that to simplify the results, as a consequence of the smallness of the radius of a well compared with the distance between the wells, the openings at the entrance and exit of the water in the stratum are assumed to be points. The small error in $\tau(\chi)$ introduced by this assumption can easily be estimated by changing the limits of integration in (9).

The mean calorimetric water temperature at the exit from the stratum θ^* is given by the expression

$$\theta^*(\bar{t}) = \left(1 - \frac{2\chi^*(\bar{t})}{\pi}\right) + \frac{2}{\pi}\int\limits_0^{\chi^*(\bar{t})} \operatorname{erf}\left(\frac{\sqrt{\mathrm{Pe}_m}\,\tau(\chi)}{\mathrm{Pe}_s\sqrt{\bar{t}-\tau(\chi)}}\,d\chi\right) \tag{10}$$

where $\chi^*(\bar{t})$ is the angle from the inner part of which to the working well the water in the stratum is admitted (it is determined as a function inverse to $\tau(\chi)$).

The first term in (10) describes the change in the temperature due to the nonequilibrium of the motion of the thermal convective front from the pumping well to the operational well, and the second describes the heat inflow from the rock mass.

We will first consider the operation of a linear battery of alternately pumping and working wells (Fig. 2a). A filtering cell is a strip of width l with an equipowered source and a drain with a discharge $\pm Q/2$. The mapping has the form

$$W = \frac{\bar{l}}{\pi}\ln\frac{1+\omega}{1-\omega}. \tag{11}$$

Taking (11) into account, we obtain from (9)

$$\tau(\chi) = \frac{4\bar{l}^2\chi}{\pi C\sin 2\chi}. \tag{12}$$

For a ring battery of radius R consisting of one pumping and n operating wells (or n pumping and one operating well) (Fig. 2b), the filtering cell is a sector with a span of $2\pi/n$ and a source and drain of discharge $\pm Q/n$. Mapping gives the formula

$$W = \bar{R}\left(\frac{(i\omega)^2}{1-(i\omega)^2}\right)^{1/n}, \tag{13}$$

whence

$$\tau(\chi) = \frac{2\pi\bar{R}}{n^2C}\int\limits_0^\infty \left(\frac{r^2}{(1+2r\cos 2\chi + r^2)}\right)^{\frac{1-n}{n}} \frac{r\,dr}{(1+2r\cos 2\chi + r^2)^2}. \tag{14}$$

For the special case when n = 2, we have from (14)

$$\tau(\chi) = \frac{\pi\bar{R}^2(1-\cos 2\chi)}{2C\sin^2 2\chi}. \tag{15}$$

As the third example, we will consider a system of linear batteries of alternately forcing and operating wells (Fig. 2c). Mapping of the filtering cell, representing a rectangle of dimensions $l \times m$, is carried out by an elliptic integral of the first kind [6]:

$$W = AF(\omega, n) = A\int\limits_0^\omega \frac{d\omega}{\sqrt{(1-\omega^2)(1-n^2\omega^2)}}, \tag{16}$$

where the constants A and n are found from the relations

$$A = l/K(\sqrt{1-n^2}),\quad K(\sqrt{1-n^2})/K(n) = 2\sqrt{m/l}$$

(K(n) is the complete elliptic integral of the first kind).

Substituting (16) into (9) we obtain

$$\tau(\chi) = \frac{\pi A^2}{C}\int\limits_0^\infty \frac{r\,dr}{\sqrt{(1+2r^2\cos 2\chi + r^4)(1+2n^2r^2\cos 2\chi + n^4r^4)}}. \tag{17}$$

As $m/l \to \infty$, expression (17) reduces to the solution for a single battery (12).

Figure 3 shows the results of a calculation of the water temperature at the exit from the stratum calculated from Eq. (10) for a single linear battery (the dashed lines), a ring battery with n = 2 (the continuous lines), and a system of linear batteries (the dash–dot lines), when $\tau(\chi)$ is given by Eqs. (12), (15), and (17),

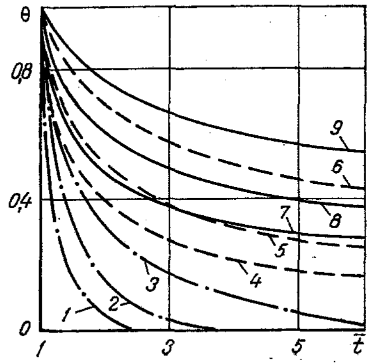

Fig. 3. Variation of the temperature of the water at the exit from the stratum: 1) $\varepsilon = 0$; 2) $\varepsilon = 0.125$, $Q = 1000 m^3/h$, $t_0 = 3.5$ years; 3) $\varepsilon = 0.395$, $Q = 100 m^3/h$, $t_0 = 35$ days; 4) $\varepsilon = 0$, 5) $\varepsilon = 0.125$; $Q = 1000 m^3/h$, $t_0 = 4$ years; 6) $\varepsilon = 0.395$, $Q = 100 m^3/h$, $t_0 = 40$ days; 7) $\varepsilon = 0$; 8) $\varepsilon = 0.125$, $Q = 1000 m^3/h$, $t_0 = 5$ days; 9) $\varepsilon = 0.395$, $Q = 100 m^3/h$, $t_0 = 50$ days.

respectively. We chose as the time scale the time of motion of the thermal front along the shortest current line, i.e., $\bar{t} = t/t_0$, where $t_0 = 2hl^2\tau(\chi = 0)/Q$. The calculations were carried out for different discharges under the following conditions: $h = 100$ m, $l = r = m = 300$ m, $\rho_m c_m = 3 \cdot 10^6$ J/m$^3 \cdot$deg, $m\rho_w c_w + (1 - m)\rho_s c_s$ $3.2 \cdot 10^6$ J/m$^3 \cdot$deg, and $\lambda_m = 2.5$ W/m$\cdot$deg. Lines 1, 4, and 7 were constructed ignoring the conductive flow of heat from the surrounding rock mass. The contribution of the thermal flow from the rock mass to the change in temperature at the exit from the stratum increases as the parameter $\varepsilon = \sqrt{Pe_m}\bar{l}/Pe_s\sqrt{C}$ increases, characterizing the ratio of the conductive to the convective heat fluxes. In addition, an increase in the number of wells in the stratum leads to a sharper drop in temperature, which is due to the reduction in the volume of the stratum and of the surrounding rock mass processed by a single well.

Note that the method described above for calculating the heat exchange in geothermal strata can be applied not only to the cases considered but to other systems of well arrangements.

NOTATION

x, y, z, coordinates; t, time; u_x and u_y, velocity components; p, pressure; ψ, current function; T_s, stratum temperature; T_m, temperature of the rock mass; h, stratum thickness; Q, well discharge; μ, coefficient of dynamic viscosity; k, permeability; m, porosity; $\rho_s c_s$, density and heat capacity of the stratum rock; $\rho_w c_w$, water density and heat capacity; ρ_m, l_m, λ_m, density, heat capacity, and thermal conductivity of the rock mass; u and s, dimensionless velocity and length of the arc along the current line. The dimensionless variables are $\bar{x} = x/h$; $\bar{y} = y/h$; $\bar{z} = z/h$; $\bar{t} = Qt/h^3$; $\bar{u}_x = u_x h^2/Q$; $\bar{u}_y = u_y h^2/Q$; $\bar{p} = pkh/\mu Q$; $\bar{\psi} = \psi h/Q$; $\bar{l} = l/h$; $\bar{R} = R/h$; $\theta = (T_s - T_0)/(T_i - T_0)$; $\bar{T} = (T_m - T_0)/(T_i - T_0)$; $\eta(\xi < 0) = 0$; $\eta(\xi > 0) = 1$; $Pe_m = Q\rho_m c_m/h\lambda_m$, and $Pe_s = Q(m\rho_w c_w + (1 - m)\rho_s c_s)/h\lambda_m$ is the Peclet number for the rock mass and stratum.

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